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On Some Fixed Point Theorems for Extension of Banach Contraction Principal In compact Metric spaces

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ABSTRACT: In the present paper, we have established some fixed point theorems in Compact metric spaces for integral type mappings. Our results are extension of famous known result Banach contraction principal. Specially we will establish some fixed point and common fixed point theorems for rational expressions in Pseudo Compact Tichonov Spaces. The results are stronger than Soni [11,12], Khan and Sharma [2,] Jain and Dixit [7] and Bohare, [3].

Keywords: Fixed Point, Fixed Point Theorem, Metric Space,

I. INTRODUCTION AND PRELIMINARIES

There are several genralizations of classical contraction mapping theroem of Banach [1]. In 1961 Edelstein [4] established the existence of a unique fixed point of a self map T of a compact metric space satisfying the inequality d(T(x), T(y)) <d (x, y)

which is genralization of Banach. In the past few years a number of authers such as Fisher [5], Soni [12] have stablished a number of intresting results on compact metric spaces. More recently Fisher and Namdeo [6], Popa and Telci [10], Sahu [13] described some valuble results in compact metric spaces.

Jain and Dixit [7], Pathak [9], Khan, S. and Sharma [8] worked on pseudo-compact Tichonov spaces.

In 2002, A. Branciari [2] analyzed the existence of fixed point for mapping f defined on a complete metric space X, d satisfying a general contractive condition of integral type

Theorem 2.2 (Branciari). Let X, d be a complete metric space, c 0,1 and let f: X → X be a mapping such that for each x, y ∈ X,

$$\int_0^{d(f_x,f_y)} \xi(t)dt \leq c \int_0^{d(f_x,f_y)} \xi(t)dt \quad 2.2.1$$

Where ξ: [0, +∞) → [0, +∞) is a Lesbesgue integrable mapping which is sum able on each compact subset of [0, +∞) , non negative, and such that for each $\epsilon > o$, $\int_0^{\epsilon} \xi(t)dt$, then f has a unique fixed point a X such that for each x X, $\lim_{n\to\infty} f^n x = a$.

After the paper of Branciari, a lot of a research works have been carried out on generalizing contractive conditions of integral type for different contractive mappings satisfying various known properties. A fine work has been done by Rhoades [11] extending the result of Brianciari by replacing the condition [2.2.1] by the following

$$\int_0^{d(f_x,f_y)} \xi(t)dt \leq \int_0^{maxd\left\{d(x,y),d(x,fx),d(y,fy),\frac{d(x,fy)+d(y,fx)}{2}\right\}} \xi(t)dt$$

In [9] the author proved the following

Theorem 2.2.3(9) Let (X, d) be a complete metric space and f : X → X such that

$$\int_0^{d(f_x,f_y)} u(t)dt \leq \alpha \int_0^{\{d(x,fx)+d(y,fy),\}} u(t)dt + \beta \int_0^{d(x,y)} u(t)dt + \gamma \int_0^{max\{d(x,fy)+d(y,fx),\}} u(t)$$

For each x, y ∈ X with non negative reals α, β, γ such that $2\alpha + \beta + 2\gamma < 1$, where u: [0, + ∞) →[0, +∞) is a Lebesgue integrable mapping which is sum able, non-negative and such that for each $\epsilon > 0$, $\int_0^{\epsilon} u(t)dt > 0$. Then f has a unique fixed point in X.

There is a gap in the proof of the theorem 2.2.3.In fact; the authors [9] used the inequality $\int_0^{a+b} u(t)dt \leq \int_0^{a} u(t)dt + \int_0^{b} u(t)dt \; for \; 0 \leq a < b,$ This is not true in general.

Also we are using effect $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$ which is only true when f (2a – x) = f(x), there is again a gap, to complete our proof that $\int_0^{kf(x)} \phi(t)dt = k\int_0^{f(x)} \phi(t)dt$ which is not true in general.

We are finding some fixed point theorems in pseudo-compact Tichonov spaces.

2.1. A topological space X is said to be Pseudo-Compact space if and only if every real valued continuous function on X is bounded.

2.2. Every Compact space is Pseudo- Compact space but converse is not true. Engleking, K.[1].

However, in metric spaces the notions 'Compact' and 'Pseudo- Compact' coincides.

2.3. A completely regular Hausdorff Space is Tichonov Space.

2.4. Product of two Tichonov Spaces is a Tichonov Space. Whereas the product of two Pseudo-Compact spaces need not be a Pseudo-Compact space.

II. MAIN RESULTS

THEOREM (3.1): Let P be a pseudo compact Tichonov space and μ be a non-negative real valued continuous function over (P x P) satisfying.

(3.1 a) μ (x, x) = 0, for all x ε P and

μ (x, y) = μ (x, z) + μ (z,y) for all x, y and z ε P.

Let T: P $\longrightarrow$ P is a continuous map satisfying;

$$\int_0^{\mu(Tx,Ty)} \Psi(t) \quad dt < \int_0^{\mu(x,y)K} \Psi(t) \quad dt \; ...(3.1b)$$

For all distinct x, y εP, with x≠y. Where Ψ : [0, +∞) → [0, +∞) is a Lesbesgue integrable mapping which is sum able on each compact subset of [0, +∞) , non negative, and such that for each ϵ > o, $\int_0^{\epsilon} \Psi(t)\, dt$

Then T has a unique fixed point in P.

Where K = $\left[1+\frac{\mu(Tx,x)\mu(Ty,x)}{\mu(x,y)}+\frac{\mu(Tx,x)\mu(Ty,x)}{\mu(Tx,x)+\mu(Ty,x)} + \frac{\mu(y,Tx)\mu(Tx,x)\mu(Ty,x)}{\mu(y,Tx)+\mu(Tx,x)+\mu(Ty,x)} \right]$

PROOF: We define d: P $\longrightarrow$ R by

ϕ (p) = μ (Tp, p)

For all p ε P, where R is the set of real numbers clearly ϕ is continuous, being the composite of two functions T andμ, since P is pseudo compact Tichonov space; every real valued continuous function over P is bounded and attains its bounds. Thus there exists a point say

Vε P, such that

ϕ (v) = inf {ϕ (p): p ε P}

It is clear that ϕ (p) C R. We now affirm that v is a fixed point for T. If not, let us suppose that Tv ≠ v. So

$$\int_0^{\phi(T(v))} \Psi(t)\, dt = \int_0^{\mu(T^2 v, Tv)} \Psi(t)\, dt$$

$$\int_0^{\phi(T(v))} \Psi(t)\, dt \prec \int_0^{\mu(Tv,v)\left[1+\frac{\mu(T(Tv),Tv)\mu(Tv,Tv)}{\mu((Tv),v)}+\frac{\mu(T(Tv),Tv)\mu(Tv,Tv)}{\mu(T(Tv),TV)+\mu(Tv,Tv)} + \frac{\mu(T(Tv),Tv)\mu(Tv,Tv)\mu(T(Tv),v)}{\mu(T(Tv),Tv)+\mu(Tv,Tv)+\mu(T(Tv),v)} \right]} \Psi(t)\, dt$$

$$= \int_0^{\mu(Tv,Tv)} \Psi(t)\, dt$$

$$\int_0^{\phi(T(v))} \Psi(t)\, dt = \int_0^{\mu(T^2 v, Tv)} \Psi(t)\, dt < \quad = \int_0^{\mu(Tv,Tv)} \Psi(t)\, dt$$

A contradiction, so T (v) = v

I.e. v εP is a fixed point for T.

To prove the uniqueness of v, if possible, let w ε P be another fixed point for T, i.e.

T w = w and w ≠ v. So

$$\int_0^{\mu(v,w)} \Psi(t)\,dt = \int_0^{\mu(Tv,Tw)} \Psi(t)\,dt$$

$$< \int_0^{\mu(v,w)} \left[\begin{array}{l} 1+\frac{\mu(Tv,v)\mu(Tw,v)}{\mu(w,v)}+\frac{\mu(Tv,v)\mu(Tw,v)}{\mu(Tv,v)+\mu(Tw,v)} \\ +\frac{\mu(w,Tv)\mu(Tv,v)\mu(v,w)}{\mu(w,Tv)+\mu(Tv,v)+\mu(v,w)} \end{array} \right] \Psi(t)\,dt$$

$\int_0^{\mu(v,\ w)} \Psi(t)\,dt < \int_0^{\mu(v,w)} \Psi(t)\,dt$ .Again it is a contradiction. Hence v ε P is unique fixed point for T in P.

This completes the proof.

THEOREM(3.2.2) : Let P be a pseudo compact Tichonov space and μ be a non-negative real valued continuous function over (P x P) .If S and T are two continuous self mapping of p satisfying:

(3.1. a) ST=TS

$$\int_0^{\mu(STx,Sy)} \Psi(t)\,dt < \int_0^{\mu(y,Tx)} \left[\begin{array}{l} 1+\frac{\mu(Tx,STx)\mu(Tx,Sy)}{\mu(Tx,y)}+\frac{\mu(Tx,STx)\mu(Tx,Sy)}{\mu(Tx,STx)+\mu(Tx,Sy)} \\ +\frac{\mu(Tx,STx)\mu(STx,y)\mu(Sy,Tx)}{\mu(Tx,STx)+\mu(STx,y)+\mu(Tx,Sy)} \end{array} \right]) \Psi(t)\,dt$$

For all distinct x, y εP, with x≠y, Where $\Psi : [0, +\infty) \to [0, +\infty)$ is a Lesbesgue integrable mapping which is sum able on each compact subset of [0, +∞) , non negative, and such that for each

$\epsilon > o$, $\int_0^{\epsilon} \Psi(t)\,dt$ Then T and S has a unique fixed point.

PROOF: We define d: P $\longrightarrow$ R by

φ (p) = μ (STp,Tp)

for all p εP, where R is the set of real numbers clearly φ is continuous, being the composite of continuous functions T ,S and μ, since P is pseudo compact Tichonov space, every real valued continuous function over P is bounded and attains its bounds. Thus there exists a point say

v ε P, such that

φ (v) = inf {φ (p): pεP}

It is clear that φ (p) C R. We now affirm that v is a fixed point for S. If not, let us suppose that Sv ≠v

$$\int_0^{\Phi(Sv))} \Psi(t)\,dt = \int_0^{\mu(STSv,\ TSv)} \Psi(t)\,dt$$

$$< \int_0^{\mu(T(v),TS(v))} \left[\begin{array}{l} 1+\frac{\mu(TS(v),STS(v))\mu(ST(v),TS(v))}{+\mu(T(v),TS(v))}+\frac{\mu(TS(v),STS(v))\mu(ST(v),TS(v))}{\mu(TS(v),STS(v))+\mu(ST(v),TS(v))} \\ +\frac{\mu(TS(v),STS(v))\mu(ST(v),TS(v))\mu(TS(v),ST(v))}{\mu(TS(v),STS(v))+\mu(ST(v),TS(v))+\mu(TS(v),ST(v))} \end{array} \right] \Psi(t)\,dt$$

$$= \int_0^{\mu(Tv,\ TSv)} \Psi(t)\,dt$$

i.e. $\int_0^{\mu(Tv,\ TSv)} \Psi(t)\,dt < \int_0^{\mu(Tv,\ TSv)} \Psi(t)\,dt$

Or φ (Sv) < φ (v)

a contradiction , v εP is a fixed point for S.

i.e. S(v)= v

But, ST= TS so

ST (v) =TS (v) =T (V)

Now we shall prove that T (v) = v.

If possible let T (v) ≠ v then

$$\int_0^{\mu(Tv,\ v)} \Psi(t)\,dt = \int_0^{\mu(ST(v),\ Sv)} \Psi(t)\,dt$$

$$< \int_0^{\mu(v,T(v))} \left[\begin{array}{l} 1+\frac{\mu(Tv,STv)\mu(Tv,S(v))}{\mu(v,Tv)}+\frac{\mu(Tv,STv)\mu(Tv,S(v))}{\mu(Tv,STv)+\mu(Tv,S(v))} \\ +\frac{\mu(Tv,STv)\mu(Tv,S(v))\mu(v,ST(v))}{\mu(Tv,STv)+\mu(Tv,S(v))+\mu(v,ST(v))} \end{array} \right] \Psi(t)dt$$

$\int_0^{\mu(\mathrm{Tv},\,\mathrm{v})} \Psi(t)\,dt < \int_0^{\mu(\mathrm{Tv},\,\mathrm{v})} \Psi(t)\,dt$ This is a contradiction. Hence v ε P is fixed point for T, i.e. T(v) =v

Uniqueness: Can be proved as Theorem 3.1.

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